Exercises for the Lecture
Techniques in Artificial Intelligence

Probability Theory and Bayesian Networks

1) Conspiracy theory

Typical for conspiracy theorists is that they accept every kind of evidence as a confirmation of the conspiracy theory. A conspiracy theorist interviews someone connected with a mysterious event. If the subject agrees that there was a conspiracy, then that of course supports the conspiracy theory. If the subject denies it, then clearly he must be part of the cover-up, thus further proving the existence of the conspiracy.

Show that, in Bayesian probability theory, it is not possible for both $E$ and $\neg E$ to be evidence in favor of hypothesis $H$. That is, it is not possible that both $P(H|E) > P(H)$ and that $P(H|\neg E) > P(H)$.

2) Would it be rational for an agent to maintain the three beliefs $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \lor B) = 0.5$? If this is the case, which probability range is rationally to be assumed for $P(A \land B)$?

3) Two students $A$ and $B$ both attend a particular lecture. Student $A$ is present on 80% of all the lecture dates, student $B$ on 60%. Their presence or absence can be regarded as independent of each other.

(a) What is the probability that, on a particular lecture date, none of the two students will be present?

(b) What is the probability that at least one of them is present on a particular date?

Assume there is another student $C$ who is present if and only if at least one of $A$ and $B$ is present.

(c) Is the presence of $A$ still independent of the presence of $B$?

(d) Construct a Bayesian Network that uses the three boolean variables $A$, $B$ and $C$ (representing the respective students) in order to represent the interrelations described above. Provide the conditional probability tables as well.

(e) Is $A$ conditionally independent of $B$ given $C$?

(f) Assume that $C$ is present on a particular date. What is the probability of $A$ being present given that $B$ is present, too?